CSCI 1311: (non-comprehensive) Final Review Worksheet

29 Apr. 2020

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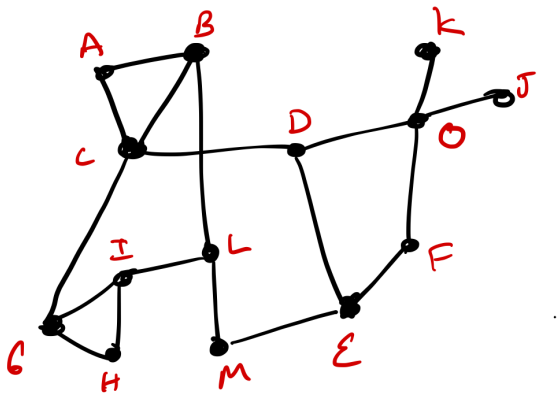
1. Consider the following graph G below



(a) Describe the set V and E for the graph?

(b) Create a matrix representation for the graph?

(c) What are the graphs articulation point(s)?

(d) Draw the minimum spanning tree using DFS starting from vertex D? Break ties by alphabetic ordering.

(e) What was the ordering of the DFS traversal that produce the spanning tree above?

(f) Draw the minimum spanning tree using BFS starting from vertex D? Break ties by alphabetic ordering.

(g) What was the ordering of the BFS traversal that produce the spanning tree above?

(h) Using the BFS, find the radius of each vertex?

(i) What is/are the center(s) of the graph?

(j) Find a subgraph G' of G that has an Euler Circuit that contains the maximum number of vertices? Draw it and describe why it has a Euler Circuit.

(k) Find a subgraph G'' of G that has an Euler Trail that contains all the vertices of G? Draw it and describe why it has a Euler Trail.

2. Prove, using induction on the number of vertices $n >= 2$ in the tree, that if you remove any edge from a tree T, you get a forest of two Trees, T_1 and T_2.

3. Prove, using induction on the height of a tree, that a full/complete binary tree T with height $h \geq 0$, that there are an odd number of internal nodes.

4. Consider the Boolean algebra of digital logic. In that language, we add the $\odot$ operator with the following truth table

x	y	$x \odot y$
0	0	1
0	1	0
1	0	1
1	1	0

   Define the Relation $x\, R\, y$ if, and only if, $x \odot y = 1$

   (a) Prove, or provide a counter example, that the $\odot$ operator is reflexive.

   (b) Prove, or provide a counter example, that the $\odot$ operator is symmetric.

5. Using the same truth table as above for $\odot$

   (a) What is a DNF formula for $\odot$?

   (b) What is a CNF formula for $\odot$?

   (c) Use equivalence statement to show that CNF and DNF for $\odot$ are equivalent.

6. Consider a machine that takes in votes from a set of judges. Judges either vote for (1) or against (0). The machine produces an affirmation (1), when there is a lone dissent, either $2/3$ in favor, one against, or $2/3$ in against, and one in favor.

   (a) Create a truth table for the judges.

   (b) Use a K-map to find a simplified statement for the judges functions.