CSCI 1311: Midterm 1 Review Worksheet

10 Feb. 2020

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1. Let the universe U be the set of GW Students. Let T be the set of GW students living in Thurston Hall, S be the set of students majoring in computer science, and V all the set of students in SEAS. Write the English sentences in set-notation and the set-notation in English
   (a) Set of CS majors that also live in Thurston Hall.
   (b) Set of GW students not in SEAS.
   (c) Set of CS majors not living in Thurston Hall.
   (d) $T \cup V$
   (e) $T \cap V$
   (f) $(T \cup S \cup V)^C$
2. Consider the following sets

$$A = \{\emptyset, 2, 5, 9\}$$
$$B = \{x \in \mathbb{Z} \mid 2 \leq x \leq 13 \wedge \text{ Prime(x)}\}$$
$$C = \{42, 64, 128\}$$

   (a) In set roster, what is $A \cap B$?
   (b) In set roster, what is $A \cup B$?
   (c) In set roster, what is $\mathcal{P}(C)$?
   (d) What is the value of $|A|$?
   (e) True/False: $\emptyset \subseteq C$
   (f) True/False: $\emptyset \subseteq A$
   (g) True/False: $\{\emptyset\} \subseteq A$
   (h) In set roster, what is $B - A$?
3. Let $P(x, y)$ be true if person x and person y are friends and $x \neq y$. Let $Q(x)$ be true if person x goes to GW. Convert the English statements to quantified, prepositional logic.
   (a) There are three people who are friends with each other..
   (b) Every person has at least one friend
   (c) There is a person who is friends with everyone.
   (d) Every person has no more than two friends.
4. Draw a truth table for the following logical expressions
   (a) $p \vee q \wedge r$

   (b) $p \to (q \vee \neg r)$

5. Prove the following with equivalent statments
   (a) $(p \wedge q) \to r \equiv (p \to r) \vee (q \to r)$
   (b) $(p \wedge q) \vee (r \wedge \neg q) \equiv (p \vee r) \wedge (p \vee \neg q) \wedge (q \vee r)$
6. Prove the following statements
   (a) If $a|b$, then $a^2|b^2$.
   (b) All primes $p > 2$ can be written as $3q + 1$ or $3q - 1$.
7. Proove by contradiction
   (a) The square root of an irrational number is irrational.
8. Prove by contrapositive
   (a) If $m + n$ is even, then either m and n are both even or m and n are both odd.
9. Prove the following using weak induction.
   (a) For all $n \geq 1$, $\sum_{i=1}^{n} 4(i + 1) = 2n(n + 3)$
   (b) For all integers $n \geq 1$, $6|(7^n - 1)$
10. Prove the following using strong induction.
    (a) For all integers $n \geq 8$, there exists integers a and b such that $n = 4a + 5b$.