CSCI 1311: Midterm 2 Review Worksheet

3 Apr. 2020

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1. Solve the recurrence relation such that you can write t_n in terms of n alone.
   (a) $t_n = t_{n-1} + 1 \qquad t_0 = 1$
   (b) $t_n = 5t_{n-1} + 4 \qquad t_0 = 11$
   (c) $t_n = t_{n-1} + 2n \qquad t_0 = 2$
   (d) $t_n = 10t_{n-1} - 21t_{n-2} \qquad t_0 = 4 \quad t_1 = 20$
2. Use induction to prove your solution to the above recurrences (a) and (c).
3. Consider the following sets

$$A = \{1, 2, 3\}$$
$$B = \{w, x, y, z\}$$
$$c = \{\alpha, \beta\}$$

   (a) Let $f : A \to \mathbb{R}$ where $f(x) = \sqrt{x}$. Is f a well-defined function?
   (b) Let $f : A \to \mathbb{R}$ where $f(x) = |\sqrt{x}|$, where $|\cdot|$ is the absolute value.
      i. Is f a well defined function?
      ii. What is the domain and co-domain of f? Explain.
      iii. Is the function one-to-one? Explain.
      iv. Is the function onto? Explain.
   (c) Let $g : A \to B$
      i. Is it possible for g to be one-to-one? If so, provide a one-to-one mapping.
      ii. Is it possible for g to *not* be one-to-one? If so, provide a non one-to-one mapping from A to B.
      iii. Is it possible for g to be onto? If so, provide a mapping that is onto, and if not, explain why not.
   (d) How many well defined functions exist for $h : C \to A$?
4. For the following functions, are they well defined, and if so are they either one-to-one and/or onto, or provide a counter example for either.
   (a) $f : \mathbb{Z} \to \mathbb{Q} : f(x) = \frac{x}{2}$
   (b) $g : \mathbb{Z} \times (\mathbb{Z} - \{0\}) \to \mathbb{Q} : g(a, b) = \frac{a}{b}$
5. Show that the cardinality of $\mathbb{Z} - \{0\}$ is countable.
6. Prove the following are equivalence relation, or find a counter example.

(a) $(\forall a, b \in \mathbb{R}^+)(a \, R \, b \iff 0 < ab \leq 1)$
(b) $(\forall a, b \in \mathbb{Z}^+)(a \, S \, b \iff a \mid b)$

7. Are any of the above relations partial-order relations?

8. Consider the partial order relation over sets A and B

$$(\forall A, B \in \mathcal{P}(\{1, 2, 3\}))({}_A R_B \iff A \subseteq B)$$

prove that it is not a total order relation.

9. Prove that the following modular equivalence is an equivalence relation:

$$(\forall a, b \in \mathbb{Z})(a \, R \, b \iff a \equiv b \pmod 5)$$

10. Find the multiplicative inverse modulo 11 to the following values
    (a) 3
    (b) 4
    (c) 5

11. Consider choose passwords with only values 1-5 and A-E, there are ten total
    (a) How many 4-length password, without repetitions exist?
    (b) If you were to choose a random 4-length password without repetitions, what is the likelihood that it begins with a 1 and a 5?
    (c) How many 4-length passwords, with repetitions, exists that contain at least a 1 and an A
    (d) How many 4-length passwords, with repetition, exist that start with an 1 and end with an A, and have at least a 3 as one of the other two digits?

12. Consider a bucket of 5 balls: 3 balls are red and 2 balls are blue. You draw 3 balls at random.
    (a) How many ways can 3 balls be drawn?
    (b) What is the likelihood of all 3 balls being red?
    (c) What is the likelihood of 2 red balls and 1 blue ball
    (d) If you were told that the first ball is blue, what is the likelihood the remaining two balls are red?