CSCI 1311: Midterm 1

13 Feb 2020

Name: ________________________ **email:** ________________________

Question Weighting

Question:	1	2	3	4	5	6	Total
Points:	7	9	9	7	4	14	50
Bonus Points:	2	2	3	0	3	5	15

Instructions

- Answer the questions in the spaces provided on the question sheets. **If you run out of room for an answer, continue on the last page of this exam, but mark in the answer space that you have done so** Show all work, but please be legible. *Microscopic writing will not be graded*
- The exam is graded out of 50 points. There are 15 bonus points available The max grade you can receive is a 65/50.
- You are allowed a handwritten notes on a single page of double sided of 8.5″x11″ paper to use on the exam. You must present your cheat sheet when you turn in your exam for inspection.
- You are not allowed to use calculators or other aides on this exam.
- Your phone should be placed in your bag, and not on your person.

1. Let the universe U be the set of people within the District of Columbia at 3:45 PM on 13 February, 2020. Let the set G be the set of GW students in that universe, the set M be the set of people on the Metro in that universe, and the set A_n be the set of people at age n or greater in that universe.

   Convert the equivalent English statement in set notation using union, intersection, set difference, or compliment, or covert the set notation in plain English

   (a) **[1 point]** The set of GW student 21 and older.

   (b) **[1 point]** The set of people in the district who are not on the metro or are not GW students.

   (c) **[1 point]** The set of people in the district who are minors (< 18 old), enrolled in GW, and currently riding metro.

   (d) **[2 points]** $(A_{65} \cup M) \cap G$

   (e) **[2 points]** $M - G$ (or written $M \backslash G$

   (f) **[*Bonus* +2 points]** $A_{30} \cap A_{40}^c$

2. Define the following sets:

$$A = \{x \in \mathbb{Z} \mid (\exists k \in Z)(x = 2k) \wedge (x > 0) \wedge (x < 10)\}$$
$$B = \{2, 8, \{3, \emptyset\}\}$$
$$C = \{B, 1, 2, 3\}$$
$$D = \{(a, b) \mid (a, b) \in A \times B\}$$

(a) **[1 point]** What is $|B|$?

(b) **[1 point]** What is $|C|$?

(c) **[1 point]** True/False: is $\{4, \emptyset\} \subseteq C$

(d) **[1 point]** True/False: is $\emptyset \subseteq C$

(e) **[1 point]** Write out set A in set-roster notation?

(f) **[2 points]** What is $\mathcal{P}(B)$ (the power-set of B) in set-roster notation?

(g) **[2 points]** What is $\{(e, f) \in D \mid f \in C\}$ in set roster notation?

(h) **[*Bonus* +2 points]** What is $C - B$ in set roster notation?

3. Define the proposition $P(x, y)$ to be true if student x is in class y, the propositions $Q(x)$ to be true if student x is a CS major, and the proposition $R(x)$ is true if the student is a freshman. Let X be the domain of students and Y be the domain of classes. Then α refer to discrete math (this class!) and β refer to UW1020 (first year writing).

   Convert the English statement to a quantified expression using "forall" ($\forall$), "exists" ($\exists$), "and" ($\wedge$), "or" ($\vee$), "negation" ($\neg$) or "implication" ($\rightarrow$), or convert the quantified expression to plain English.

   (a) **[2 points]** There is at least one student taking discrete math.

   (b) **[2 points]** If a student is a cs major, then they are taking discrete math.

   (c) **[2 points]** All students are enrolled in at least two classes.

   (d) **[3 points]** $(\forall x \in X)(\forall y \in Y)[(P(x, y) \wedge y \neq \beta) \rightarrow \neg R(x)]$

   (e) **[*Bonus* +3 points]** $(\exists a, b, c, d, e \in Y)[(\forall x \in X)(\forall y \in Y)(P(x, y) \rightarrow y \in \{a, b, c, d, e\}]$

4. Create a truth table for the following logical statements

   (a) **[2 points]** $p \vee \neg q$

   (b) **[2 points]** $p \oplus \neg q$

   (c) **[3 points]** $p \wedge q \to r$

5. Show the following via equivalent statements. Show your work.

(a) **[2 points]** $p \rightarrow q \equiv \neg q \rightarrow \neg p$

(b) **[2 points]** $\neg(\neg p \vee q) \wedge \neg p \equiv \mathbf{c}$

(c) **[*Bonus* +3 points]** $(p \wedge \neg q) \vee (\neg p \wedge q) \equiv (p \vee q) \wedge \neg(p \wedge q)$

6. Complete the following proofs.

   (a) **[5 points]** Prove the following: for all integers n, exists an integer q, such that $n^2 = 4q$ or $n^2 = 4q+1$

(b) **[4 points]** Proof the following: if n^2 is odd, then n is odd.

(c) **[5 points]** Prove the following using (weak) induction: for all positive integers, if $n \geq 1$, then $3|(n^3 + 2n)$. (*Be sure to clearly state your base case, inductive step, and your inductive hypothesis to get full credit*

(d) **[*Bonus* +5 points]** Prove that the sum of the first n positive odd numbers are n^2.

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