CSCI 1311: Midterm 2

7 April 2020

Name: _____ **email:** _____

Question Weighting

Question:	1	2	3	4	5	6	7	8	9	10	Total
Points:	10	5	5	3	1	6	7	3	5	5	50
Bonus Points:	0	0	0	0	8	0	0	2	5	0	15

Submission Instructions

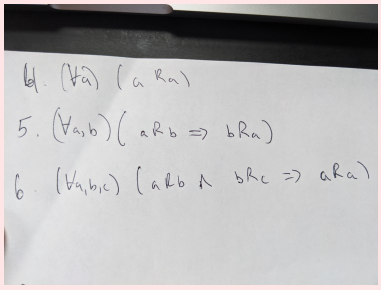
When submitting on gradescope, you can submit pictures of your answers. If you do so then …

…you should upload a zoomed in image per question/part; do not submit a single large picture of an entire page if it can be practically avoided.

This will really help improve grading. For example, if I was answering the questions

> 5. What is the definition for a relation to be symmetric?
>
> [ ]

I would upload a zoomed in picture (like the one on the right) that clearly included the answer to Question 5. Perhaps the image would include the answer to some other questions, like Question 4 or 6, but each question is well marked and zoomed in for this group of related questions. **Please do not upload an entire page of answers, which makes it very difficult to grade, for example, to identify Question 5.**



Exam Instructions

- The exam is due on Gradescope at 6:00pm EDT (3:00pm PDT). If you have DSS accommodations, it is due at 8:00pm EDT (5:00pm PDT).
- The exam is graded out of 50 points. There are 10 bonus points available. The max grade you can receive is a 65/50.
- The exam is open notes, open book, and open slides. It is not open internet or open in any other way. **You may not use piazza posts or class videos.** You are expected to only use the prescribed resources, your notes, the book, and the slides. Failure to do so could lead to a 0 on the exam, and likely failing the class.
- **YOU MAY NOT USE A CALCULATOR.** You should leave your answers in reduced formulas where appropriate
- An instructor will always be available via Zoom if there are any questions that emerge during the exam.

  `https://zoom.us/j/654707922?pwd=YzhHVmM1Ym9nSVlsSTZKaGNGT24yZz09`
  (meeting ID: 654 707 922, pass: cs1pg2)

1. Solve the following recurrences, that is, what are the formulas for t_n in terms of n alone, without any references to t_{n-1}

   (a) **[5 points]** $t_n = 2t_{n-1} + 1 \quad t_0 = 1$

   (b) **[5 points]** $t_n = 2t_{n/2} \quad t_1 = 1$
   Assume that n ***is always a power of 2, so that*** $n/2$ ***cleanly divides leaving another power of 2.***

2. **[5 points]** Solve the following second-order, linear homogeneous recurrence relation. That is, describe an answer in terms of n alone.

$$t_n = 8t_{n-1} + 20t_{n-2} \quad t_0 = 4 \quad t_1 = 16$$

3. **[5 points]** Prove that the following recurrence relations

$$t_n = t_{n-1} + 4n \qquad t_0 = 10$$

is solved as

$$t_n = 2n^2 + 2n + 10$$

4. Consider the following sets

$$A = \{42, -2, 3\}$$
$$B = \{B, G, M\}$$

(a) **[1 point]** Is the following a well-defined function? If yes, why, if no, why not?

$$f : A \to B \quad f = \{(42, B), (-2, B), (-2, M)\}$$

(b) **[1 point]** Is the following a well-defined function? If yes, why, if no, why not?

$$f : A \to B \quad f = \{(42, B), (-2, G)\}$$

(c) **[1 point]** Is the following a well-defined function? If yes, why, if no, why not?

$$f : A \to B \quad f = \{(42, B), (-2, B), (3, B)\}$$

5. Consider two sets D and C where $|D| = 5$ and the $|C| = 3$

(a) **[1 point]** How many one-to-one, well-defined functions exist for $f : D \to C$? Explain your answer and show your work.

(b) **[*Bonus* +8 points]** How many onto, well-defined functions exist for $f : D \to C$? Explain your answer and show your work. **This is trickier than it may seem. Show your work!**

6. Consider the following function:
   The floor function $\lfloor \cdot \rfloor$ ***drops a value to the smallest integer number.***
   For example $\lfloor 1/2 \rfloor = 0$***,*** $\lfloor 16/3 \rfloor = 5$***, and*** $\lfloor 3.1415926 \rfloor = 3$

$$f : (\mathbb{Z}^+ \times \mathbb{Z}^+) \to (\mathbb{Z}^+ \cup \{0\}) \qquad f(a, b) = \left\lfloor \frac{a}{b} \right\rfloor$$

(a) **[3 points]** Is the function f one-to-one? If yes, prove it, if no, provide a counter example?

(b) **[3 points]** Is the function f onto? If yes, prove it, if no, provide a counter example?

7. Consider the relation R over integers $\mathbb{Z}$.

$$(\forall a, b \in \mathbb{Z})(a \, R \, b \iff a^2 \leq b^2)$$

(a) **[1 point]** Is the relation reflexive? Prove it, or provide a counter example.

(b) **[2 points]** Is the relation symmetric Prove it, or provide a counter example.

(c) **[2 points]** Is the relation transitive? Prove it, or provide a counter example.

(d) **[2 points]** Is the relation anti-symmetric? Prove it, or provide a counter example.

8. Consider the following relation:

$$(\forall a, b \in \mathbb{R}^+)(a \, R \, b \iff \frac{a}{b} \leq 1)$$

Note that $0 \notin \mathbb{R}^+$

(a) **[3 points]** Prove that the relation is a partial ordering over $\mathbb{R}^+$

(b) **[*Bonus* +2 points]** Prove that the relation is a total ordering over $\mathbb{R}^+$.

For the following questions, Your answers should simply include the formula for each of the calculations, like x^n or $x^m - y^n$, or $x \cdot y \cdot z$ or $P(n, r)$ or $\binom{n}{r}$. You can write out the final counts if you like, but just writing a number is not sufficient for full credit.

9. Consider selecting 4 digit PINs from numbers on the interval [0,9]
Show your work for full credit!

   (a) **[1 point]** How many total PINs exist?

   (b) **[2 points]** How many PINs contain at least a 0 **or** at least a 5 **or** at least a 7?

   (c) **[2 points]** How many PINs do not contain a 7, but have at least a 0 **or** a 5?

   (d) **[*Bonus* +5 points]** How many PINs contain at least a 7 and at least a 5 and at least 0?

10. **[5 points]** In 5-card poker, how many different straights exist that begin or end with an Ace. That is, $A, 2, 3, 4, 5$ or $10, J, Q, K, A$.

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