

Dice - sixsides

rolled it twice, how many outcomes?

$$6 \cdot 6 = 36$$

multiplication rule

{A, B, C, D}

A B C D

A B D C

A D B C

⋮

1st 2nd 3rd 4th

$$4 \cdot 3 \cdot 2 \cdot 1$$

$$4 \cdot 3 \cdot 2 \cdot 1 = 24$$

Factorial

$$4! = 4 \cdot 3 \cdot 2 \cdot 1$$

$$n! = n \cdot (n-1) \cdot (n-2) \cdot \dots \cdot (1)$$

$$1! = 1$$

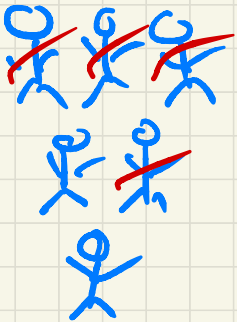
$$0! = 1$$

$$\begin{aligned}
 5! &= 5 \cdot 4! \\
 &= 5 \cdot (4 \cdot 3 \cdot 2 \cdot 1) \quad 24 \\
 &= 120
 \end{aligned}$$

$$\frac{5!}{3!} = \frac{5 \cdot 4 \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{3} \cdot \cancel{2} \cdot \cancel{1}} = 5 \cdot 4 = 20$$

Permutations

Number of arrangements of a set of items



$$\begin{aligned}
 & \frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4} = 360 \quad \begin{matrix} \cancel{2!} \\ \cancel{1} \end{matrix} \\
 &= \frac{6!}{2!} = \frac{6 \cdot 5 \cdot 4 \cdot 3 \cdot \cancel{2} \cdot \cancel{1}}{\cancel{2} \cdot \cancel{1}}
 \end{aligned}$$

r-permutation of n

n items arranged in a row of length r

$$P(n, r) \quad {}_n P_r$$

$$P(6, 4) = \frac{6!}{2!} = \frac{[6 \cdot 5 \cdot 4 \cdot 3] \cdot 2 \cdot 1}{(6-4)! = 2!}$$

$$P(n, r) = \frac{n!}{(n-r)!}$$

~~n < r?~~

$$n \geq r$$

n = r?

$$P(n, n) = \frac{n!}{(n-n)!} = \frac{n!}{0!} = n!$$

$$~~P(n, 1) = n! = \frac{n!}{n-1}~~$$

$$P(n, 1) = \frac{n!}{(n-1)!} = \textcircled{n}$$

How many 4-permutations of 7

$$P(7, 4) = \underbrace{7 \cdot 6 \cdot 5 \cdot 4}_{4! \cdot 2!} = 840$$

$$P(26, 3) = 26 \cdot 25 \cdot 24 = 15,600$$

$$\forall n \geq 2, P(n, 2) + P(n, 1) = n^2$$

$$= n \cdot (n-1) + n$$

$$= n(n-1+1)$$

$$= n(n)$$

$$= n^2$$