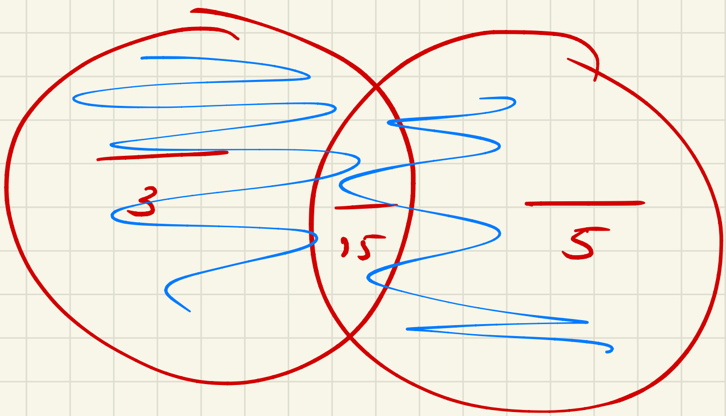
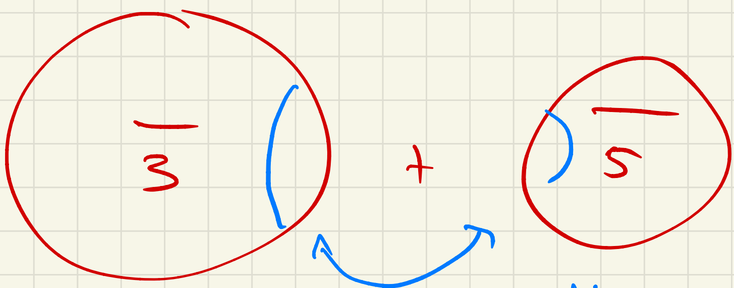
How many numbers between [1, 1000] are divisible by 3 or 5

$$\left\lfloor \frac{1000}{3} \right\rfloor = 333 \qquad \left\lfloor \frac{1000}{5} \right\rfloor = 200$$



union

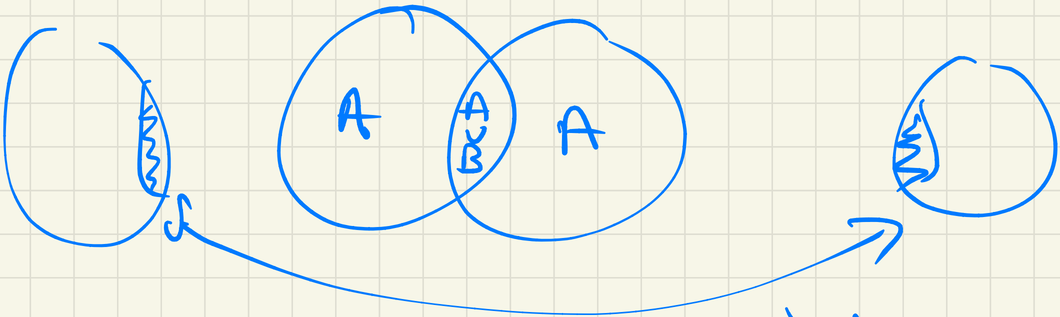
$$\left\lfloor \frac{1000}{15} \right\rfloor = 66$$



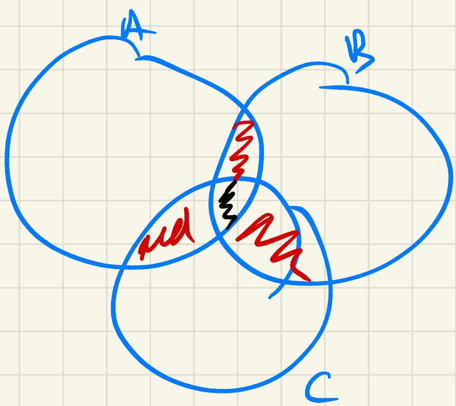
the intersection exist in both sides!

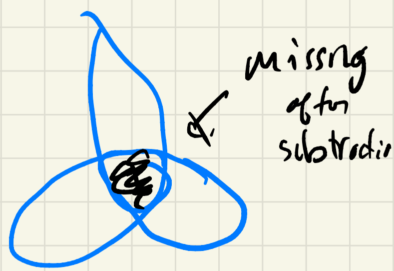
$$333 + 200 - 66 = \underline{\underline{467}}$$

$$N(A \overset{\text{or}}{\cup} B) = N(A) + N(B) - N(A \underset{\text{and}}{\cap} B)$$



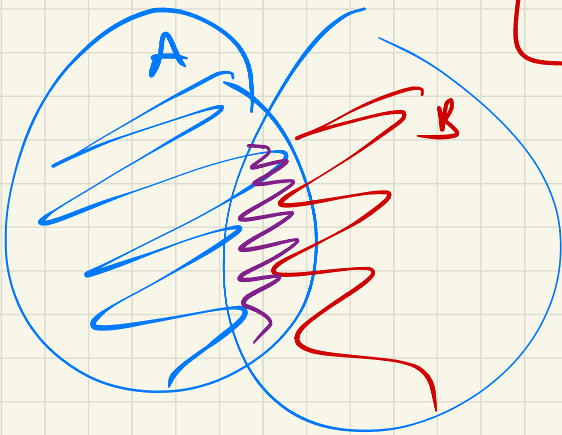
Count these twice
So subtract out!

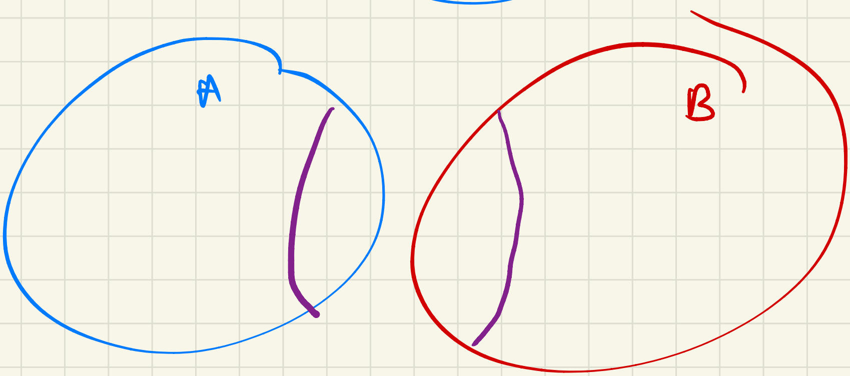




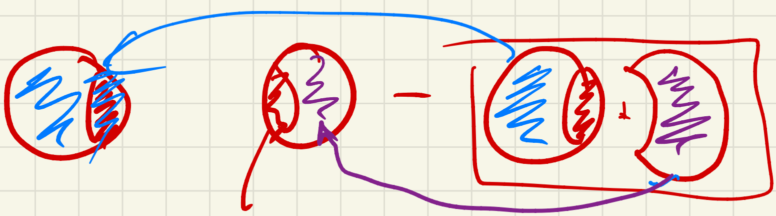
$$N(A) + N(B) + N(C) - N(A \cap B) - N(A \cap C)$$
$$- N(B \cap C) + N(A \cap B \cap C)$$

$$N(A \cap B) = N(A) + N(B) - N(A \cup B)$$





$$N(A) + N(B) - N(A \cup B$$



$= N(A \cap B)$

4 digit PINS

Have a at least 0 and a at least 1

$$N(0 \cap 1) = N(0) + N(1) - \underline{N(0 \cup 1)}$$

at least a 0: $10^4 - 9^4$

at least 1: $10^4 - 9^4$

$N(0 \cup 1)$ = at least a $\underline{0}$ or at least a $\underline{1}$

negate

$\neg$ (at least 0 $\vee$ at least 1) = $\underbrace{\text{No } 0 \wedge \text{ No } 1}_{8^4}$

or, and

$10^4 - 8^4$

$$\underbrace{(10^4 - 9^4)}_{N(0)} + \underbrace{(10^4 - 9^4)}_{N(1)} - (10^4 - 8^4) = \boxed{10^4 - 2 \cdot 9^4 + 8^4}$$