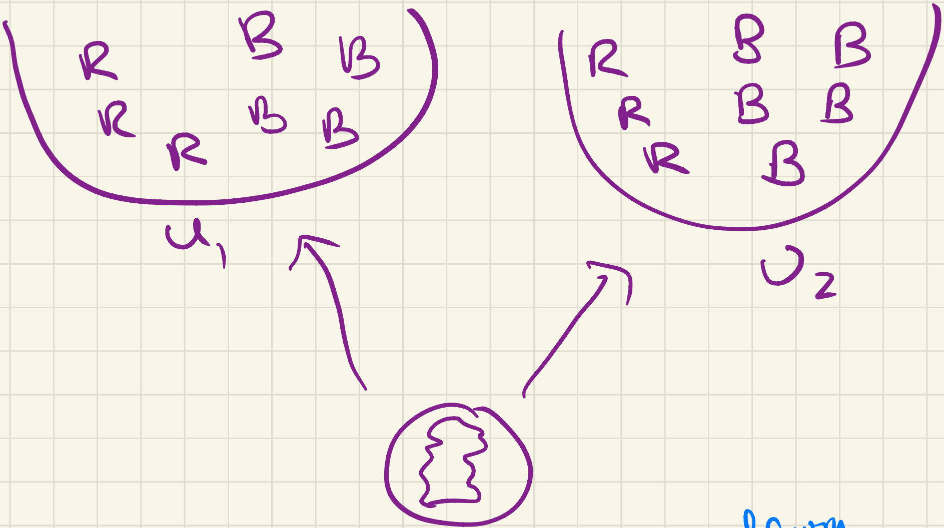


A ← drawing a blue ball

$$P(U_1 \mid \overset{A}{\cancel{B}}) = \frac{P(U_1 \cap A)}{P(A)}$$

$$P(A \mid U_1) = \underline{3/7}$$

$$P(A \mid U_2) = \underline{5/8}$$

?

Conditional Probability Formula

$$P(B|A) = \frac{P(B \cap A)}{P(A)}$$

$$\boxed{P(A)\,P(B|A) = P(B \cap A)}$$

$$\boxed{P(A) = P(U_1 \cap A) + P(U_2 \cap A)}$$

$$\boxed{P(U_1 \cap A)} = P(A \cap U_1)$$

$$= P(U_1) \cdot P(A|U_1)$$

$$= \frac{1}{2} \cdot \frac{3}{7}$$

$$= \frac{3}{14}$$

only two ways we can draw a blue ball

$$\boxed{P(U_2 \cap A)} = P(A \cap U_2)$$

$$= P(U_2) \cdot P(A|U_2)$$

$$= \frac{1}{2} \cdot \frac{5}{8}$$

$$= \frac{5}{16}$$

$$P(A) = P(A \cap U_1) + P(A \cap U_2)$$

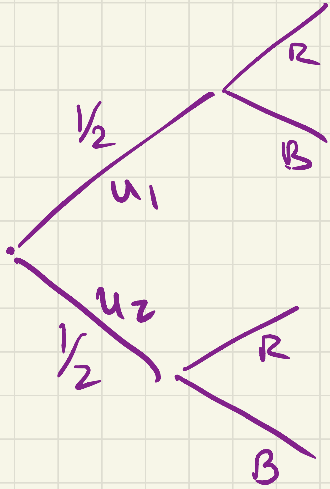
$$= \frac{3}{14} + \frac{5}{16}$$

$$= \frac{59}{112}$$

$$P(U_1 \mid A) = \frac{P(A \cap U_1)}{P(A)}$$

$$= \frac{3/14}{59/112}$$

$$\hat{=} \; 40.7\%$$



$\boxed{P(R \mid U_1) = 3/7 \cdot P(U_1) = 3/14}$

$\boxed{P(B \mid U_1) = 4/7 \cdot P(U_1) = 4/14}$

$\boxed{P(R \mid U_2) = 3/8 \; P(U_2) = 3/16}$

$\boxed{P(B \mid U_2) = 5/8 \cdot P(U_2) = 5/16}$

$$P(R) = P(R \mid U_2) + P(R \mid U_1)$$

$$= \boxed{3/16 + 3/14} = 0.4017$$

$$P(U_2 \mid R) = \boxed{\frac{3/16}{3/16 + 3/14}}$$

$$= 0.4667$$

Bayes' Rule

$$\underbrace{B_1, B_2, \dots, B_n}_{\text{choosing urns}}$$

$$\underbrace{A \text{ other event}}_{\text{drawn a blue ball}}$$

$$P(B_k \mid A) = \frac{P(A \mid B_k)\, P(B_k)}{P(A \mid B_1)\, P(B_1) + P(A \mid B_2)\, P(B_2) \dots}$$

$$P(U_1 \mid A) = \frac{P(A \cap U_1)}{P(A)} = \frac{P(A \mid U_1)\, P(U_1)}{P(A \mid U_1)\, P(U_1) + P(A \mid U_2)\, P(U_2)}$$

$P(R) = 3/6$

$P(B) = 2/6$

$P(Y) = 1/6$

$$P(R \mid 6) = \frac{P(R \cap 6)}{P(6)}$$

$$P(6) = P(6 \mid R)P(R) + P(6 \mid B)P(B) + P(6 \mid Y)P(Y)$$

$$= 1/6 \cdot 3/6 + 1/6 \cdot 2/6 + 1/6 \; 1/6$$

$$= 3/36 + 2/36 + 1/36$$

$$= 6/36 = 1/6$$

$$P(R \cap 6) = P(6 \mid R)\, P(R)$$

$$3/36$$

$$\frac{3/36}{6/36} = \frac{3}{6} = \boxed{\frac{1}{2}}$$

$$P(Y \mid I) = \frac{P(Y \cap I)}{P(I)}$$

$$P(I) = P(I \mid R)\,P(R) + P(I \mid B)\,P(B) + \boxed{P(I \mid Y)\,P(Y)}$$

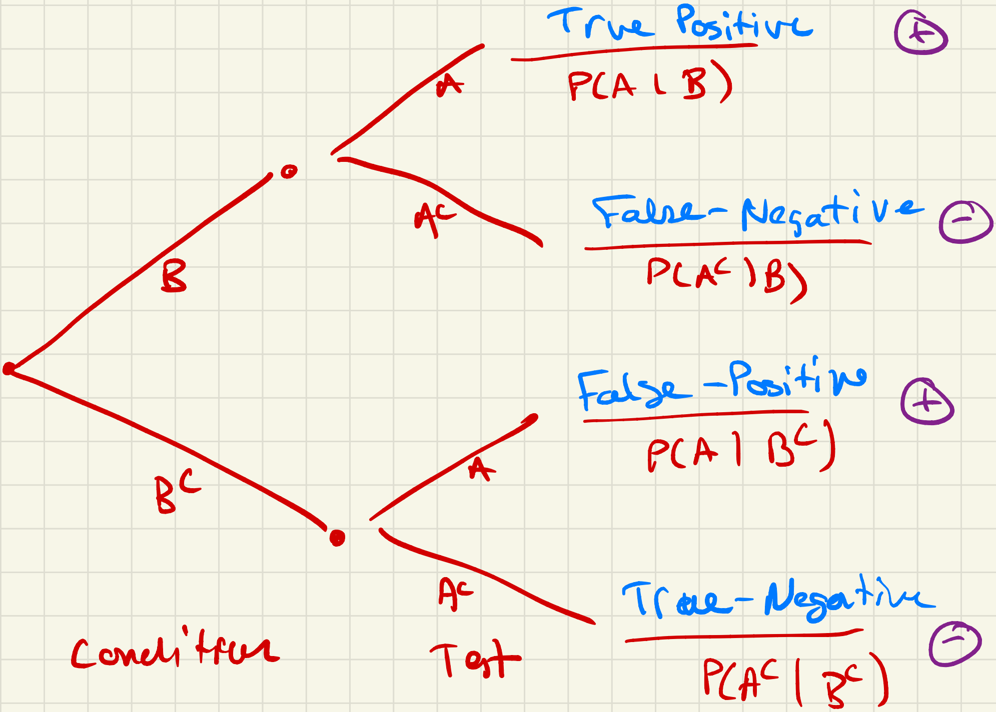
$$= \tfrac{1}{6} \cdot \tfrac{3}{6} + \tfrac{1}{6} \cdot \tfrac{2}{6} + \tfrac{1}{6} \cdot \tfrac{1}{6}$$

$$= \tfrac{6}{36} = \tfrac{1}{6}$$

$$P(Y \cap I) = P(I \mid Y)\,P(Y)$$

$$= \tfrac{1}{36}$$

$$\frac{P(Y \cap I) \to \tfrac{1}{36}}{P(I) \to \tfrac{6}{36}} = \frac{1}{6}$$



Test is A

~~Condition~~ Disease is B

$P(B) = 0.005$

$P(B^c) = 0.995$

False Positive

$P(A \mid B^c) = 0.03$

True Negative

$P(A^c \mid B^c) = 0.97$

False Negative

$P(A^c \mid B) = 0.01$

True Positive

$P(A \mid B) = 0.99$

$$P(B \mid A) = ?$$

$$P(B \mid A) = \frac{P(A \mid B)\,P(B)}{\underset{\text{True-Positive}}{P(A \mid B)\,P(B)} + \underset{\text{False-Positive}}{P(A \mid B^c)\,P(B^c)}}$$

A is the test

B is the disease

$$= \frac{(0.99)(0.005)}{(0.99)(0.005) + (\underset{0.03}{\cancel{0.97}})(.995)}$$

$$\boxed{\cong 14.2\%}$$

$$P(B^c \mid A^c)$$

$$P(B^c \mid A^c) = \frac{P(A^c \mid B^c)\, P(B^c)}{\underbrace{P(A^c \mid B^c)}_{\text{True Negative}}\, P(B^c) + \underbrace{P(A^c \mid B)}_{\text{False Negative}}\, P(B)}$$

$$= \frac{(0.97)(0.995)}{(0.97)(0.995) + (0.01)(0.005)}$$

$$= \frac{0.96515}{0.96515 + 0.00005}$$

$$= \frac{0.96515}{0.9652} \approx 0.99995$$

$$99.995\%$$