Boolean Algebra

↳ True
or
FALSE

B : set of elements

+ : conjunction "plus"

· : subjunction "product"

' : complement "not" "negation"

0 : identities
1 : under the operators

Commutative

$\cdot$ $\underline{xy}$

$(x, y \in B)$ $x + y = y + x$

$x \cdot y = y \cdot x$

Associative

$x + (y + z) = (x + y) + z$

$x(yz) = (xy)z$

Distribute

$x(y + z) = xy + xz$

$x + (yz) = (x + y)(x + z)$

Identity Law

$x + 0 = x$

$x \cdot 1 = x$

Complement

$x + x' = 1$

$x \cdot x' = 0$

$1 \quad \top \quad$ Top

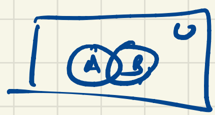
$0 \quad \bot \quad$ bottom

Boolen Algebra

B: { all preposition statements }

$\vee \quad \wedge \quad \neg \quad t \quad c$

B: 2^U (power set)



$\cup \quad \cap \quad \square^c \quad \emptyset \quad U$

$$(\forall x, a_1, a_2 \in B)(x' = a_1 \wedge x' = a_2 \implies a_1 = a_2)$$

Premise

$$x' = a_1 \qquad x' = a_2$$

x' is the complement of x

a_1 is the complement of x

a_2 is the complement of x

$$x + a_1 = 1 \qquad x \cdot a_1 = 0$$

$$x + a_2 = 1 \qquad x \cdot a_2 = 0$$

$$\begin{aligned}
\boxed{a_1} &= a_1 \cdot 1 \\
&= a_1 \cdot (x + a_2) \\
&= a_1 \cdot x + a_1 \cdot a_2 \\
&= 0 + a_1 \cdot a_2 \\
&= x \cdot a_2 + a_1 \cdot a_2 \\
&= a_2 (x + a_1) \\
&= a_2 (x + a_1) \\
&= a_2 \cdot 1 \\
&\boxed{= a_2}
\end{aligned}$$

$$x + x = x$$
$$= x + 0$$
$$= x + (x \cdot x')$$
$$= (x+x)(x+x')$$
$$= (x+x)(1)$$
$$= x + x$$

$$x + 0 = x$$
$$x \cdot 1 = x$$
$$x + x' = 1$$
$$x \cdot x' = 0$$

$$x \cdot x = x$$
$$= x \cdot 1$$
$$= x(x+x')$$
$$= xx + xx'$$
$$= xx + 0$$
$$= xx$$

$x + 1 = 1$

$(x + x) + x' =$

$x + x' =$

$1 = 1$

$\simeq$

$x \cdot 0 = 0$

$(x \cdot x) \cdot x' = 0$

$x \cdot x' = 0$

$0 = 0$

$\simeq$

$x + 0 = x$

$x \cdot 1 = x$

$x + x' = 1$

$x \cdot x' = 0$

$$x + (x \cdot y) = x$$

$$(x+x)(x+y)$$

$$x(x+y) = x$$

$$x + 0 = x$$

$$x \cdot 1 = x$$

$$x + x' = 1$$

$$x \cdot x' = 0$$

$$= x \cdot 1$$

$$= x \cdot (1+y)$$

$$= x + (xy)$$

$$= (x+x)(x+y)$$

$$= x(x+y)$$

De Morgan

$$(x+y)' = x' \cdot y'$$

$$(x \cdot y)' = x' + y'$$

$$a = (x+y) \qquad a' = (x' \cdot y')$$

$$a + a' = 1$$

$$a \cdot a' = 0$$

$$(x+y) + (x' \cdot y') = 1$$

$$x + (y + (x' \cdot y')) = 1$$

$$x + (y+x')\underbrace{(\cancel{y+y'})}_{1} = 1$$

$$x + y + x' = 1$$

$$x + x' + y = 1$$

$$1 + y = 1$$

$$1 = 1 \quad \checkmark$$

$$(x+y) \bullet (x' \cdot y') = 0$$

$$x(x' \cdot y') + y(x' \cdot y') = 0$$

$$(x \cdot x')y' + (y \cdot y')x' = 0$$

$$0y' + 0x' = 0$$

$$0 + 0 = 0$$

$$0 = 0$$