

$$B = \{1, 0\}$$

$+$  = OR

$\cdot$  = AND

$'$  = NOT

$1$  = TOP

$0$  = Bottom

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$x$	$y$	$x+y$
0	0	0
0	1	1
1	0	1
1	1	1

$x$	$y$	$x \cdot y$
0	0	0
0	1	0
1	0	0
1	1	1

$x$	$x'$
0	1
1	0

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# Boolean Expressions

- 0 or 1
- boolean  $\mathcal{X}$  stands for a 0 or 1
- e and b are expressions  
 $e, e+b, e \cdot b, e'$

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$$x+y \quad x \quad (x'+y) \cdot (z)$$

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$x/y$   $\leftrightarrow$  not greater or in  
the boolean algebra

# Boolean Funktion

$$f: \underbrace{B^n}_{B \times B \times B \times \dots \times B} \rightarrow B$$

$$f(x, y, z) = \underbrace{\hspace{10em}}_{\{0, 1\}}$$

$$x \oplus y$$

if either  $x$  or  $y$   
is 1 but not both

$x$	$y$	$(x+y)$	$(xy)'$	$((x+y)(xy)')$
0	0	0	1	0
0	1	1	0	0
1	0	1	0	0
1	1	0	1	0

# MINTERM

$$B^n \rightarrow B$$

n-boolean variabeln  
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$$x \cdot y \cdot z$$

$$x \cdot y \cdot z'$$

$$x \cdot y' \cdot z$$

$$x' \cdot y' \cdot z$$

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	$x$	$y$	$z$	
$x' \cdot y' \cdot z'$	→	0	0	0 = 1
$x' \cdot y' \cdot z$	→	0	0	1 = 1
$x' \cdot y \cdot z'$	→	0	1	0 = 1
$x' \cdot y \cdot z$	→	0	1	1 = 1
$x \cdot y' \cdot z'$	→	1	0	0 = 1
$x \cdot y' \cdot z$	→	1	0	1 = 1
$x \cdot y \cdot z'$	→	1	1	0 = 1
$x \cdot y \cdot z$	→	1	1	1 = 1

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# DNF

Disjunctive  
Normal  
Form

minterm  
canonical  
Form

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x	y	z	w	...	a		
0	0	0	0	0	0	1	1
0	0	0	0	0	1	0	0
0	0	0	0	1	1	1	1
				⋮			
1	1	1	1	1	1	1	1

# Maxterms

n wide Boolean function

the sum of all the literals

$$x + y + z$$

$$x + y' + z'$$

$$x' + y' + z$$

$$x + y + z'$$

...

CNF

Conjunctive  
Normal  
Form

Maxterm  
Canonical  
Form

$$g(x, y, z) = (x + y + z') (x' + y' + z) (x + y + z')$$

x	y	z	b	b'	(matrix)'
0	0	0	0	1	x'y'z'
0	0	1	0	1	x'y'z
0	1	0	0	1	x'y z'
0	1	1	0	0	
1	0	0	0	1	x y' z'
1	0	1	0	0	
1	1	0	0	0	
1	1	1	0	0	

$$\begin{aligned}
 b' = & \\
 & + (x'y'z')' = (x + y + z)' + \\
 & + (x'y'z)' = (x + y + z')' + \\
 & + (x'y z')' = (x + y' + z)' + \\
 & + (x y' z')' = (x' + y + z)' +
 \end{aligned}$$

$$b' = \begin{aligned} & (x' y' z')' = (x + y + z) + \\ & + (x' y' z)' = (x + y + z') + \\ & + (x' y z')' = (x + y' + z) + \\ & + (x y' z') = (x' + y + z) \end{aligned}$$

$$f(x, y, z) = (x' y' z') + (x' y' z) + (x' y z') + (x y' z')$$

$$f(x, y, z)' = \left( (x' y' z') + (x' y' z) + (x' y z') + (x y' z') \right)'$$

$$f(x, y, z) = (x + y + z) \cdot (x + y + z') \cdot (x + y' + z) \cdot (x' + y + z)$$

CNF

x	y	z	f	minutms	maxterms
0	0	0	1	$x' y' z'$	
0	0	1	0	$x' y' z$	$x + y + z'$
0	1	0	1	$x' y z'$	
0	1	1	0	$x' y z$	$x + y' + z'$
1	0	0	0	$x y' z'$	$x' + y + z$
1	0	1	1	$x y' z$	
1	1	0	1	$x y z'$	
1	1	1	1	$x y z$	

DNF

$$f(x) = (x' y' z') + (x' y z') + (x y' z) + (x y z') + (x y z)$$


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CNA

$$f(x) = (x + y + z') (x + y' + z) (x' + y + z)$$


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