# CSCI 1311: Problem Set 1

Due: 27 Jan 2020

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**Instructions**:

- Your submission must be **typed** and submitted to gradescope as a single pdf.
- You must include a cover page, that contains your name, the assignment information, the date, and your GW email address. No answers to questions should appear on the cover page.
- Try and organize your submission such that answers to questions (or parts of questions) do not span multiple pages. This will make it much easier to grade. Ideally, each page will start with a new question (or part of question). See the sample for PS0 for a nice easy formatting.
- On gradescope, be sure to mark which page your answer to each question (or sub question) is located. Doing so inaccurately could lead to issues with grading.

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**Question Weighting**

| Question: | 1 | 2 | 3 | 4 | 5 | 6 | Total |
|---|---|---|---|---|---|---|---|
| Points: | 20 | 20 | 5 | 20 | 20 | 15 | 100 |

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1. Define the following sets as:

$$\begin{aligned} A =& \{1, 3, 6, 10, 15, 21\} \\ B =& \{x \in \mathbb{Z} \mid x \in (-2, 10]\} \\ C =& \{x \in \mathbb{Z}^+ \mid \frac{x}{3} \in \mathbb{Z}\} \\ D =& \{(a, b) \in A \times B \mid a < 4\} \end{aligned}$$

(a) Fill in the $\square$ with the most appropriate symbol ($\in, \notin, \subseteq, \nsubseteq$) for each of the formulas below

i. **[1 point]** $8 \square A$

ii. **[1 point]** $8 \square B$

iii. **[1 point]** $\{3, 6, 15\} \square C$

iv. **[1 point]** $\{(3, 6)\} \square D$

v. **[1 point]** $D \square A \times B$

vi. **[1 point]** $A \square \mathcal{P}(B)$

(b) Evaluate the following sets, writing your answers in set roster notation:

i. **[1 point]** $A \cup B$

ii. **[1 point]** $A \cap B$

iii. **[1 point]** $B - A$
iv. **[1 point]** $C \cap \emptyset$
v. **[2 points]** $(A \cap C) \times A$
vi. **[2 points]** $\mathcal{P}(A \cap C)$

(c) What are sizes (cardinality) indicated below (show some work)?
i. **[1 point]** $|D|$
ii. **[1 point]** $|A \cap B \cap C|$
iii. **[1 point]** $|\mathcal{P}(A)|$
iv. **[1 point]** $|C|$

(d) Two sets are described as **disjoint** if, and only if, they have no elements in common.
i. **[1 point]** Are any two sets from $A, B,$ or $c$ disjoint?
ii. **[1 point]** Which set is always disjoint from all other sets? Why?

2. Let's define the following sets:

$$
\begin{aligned}
D_h &= \text{the set of Democrats in the house of representatives} \\
D_s &= \text{the set of Democrats in the senate} \\
R_h &= \text{the set of Republicans in the house of representatives} \\
R_s &= \text{the set of Republicans in the senate} \\
H_n &= \text{the set of representatives of age } n\text{, or below} \\
S_n &= \text{the set of senators of age } n\text{, or below} \\
M &= \text{the set of male identifying representatives and senators} \\
F &= \text{the set of female identifying representatives and senators} \\
J &= \text{the set of representatives or senators who are currently serving in their first term}
\end{aligned}
$$

There are 100 senators in the senate, and 435 representatives in the house. For the purpose of simplification (without prejudice), let's say all senators and representatives identify as either male or female and are either a Democrat or a Republican. To be a senator, you must be 35 years or older, and to be a representative, you must be 25 years or older.

(a) Describe in understandable English, each of the following sets:
i. **[2 points]** $F \cap D_h \cap H_{35}$
ii. **[2 points]** $F^c \cap J$
iii. **[2 points]** $(D_h \cup R_h) \cap J$
iv. **[2 points]** $(J^c \cap F) \cap (H_{50} \cup S_{50})^c$
v. **[2 points]** $M \cap D_s \cap (H_{45} \cup S_{45})$

(b) For each of the sentences, convert to set notation using the described sets, $\cup, \cap, -, \Box^c$ (complement), and $|\Box|$ (cardinality), as well as standard comparison relations $=, \neq, <, >, \leq, \geq$.
i. **[2 points]** The number of senators of age 45 or younger is greater than the number of representatives over the age of 75.
ii. **[2 points]** The set of congresspersons who are in their second term or greater.
iii. **[2 points]** The number of Democrats in congress who identify as female.
iv. **[2 points]** There are no senators under the age of 35.
v. **[2 points]** The number of non-first-term representatives that can run for senate.

3. Use a truth table to show the following equivalences
(a) **[2 points]** $p \vee (p \wedge q) \equiv p$

(b) **[3 points]** $p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

4. Using equivalent forms, show the following. Indicate each step and the why you can take each step (e.g., by commutative)
   (a) **[5 points]** $\neg p \lor q \equiv (p \land q) \lor (\neg p \land q) \lor (\neg p \land \neg q)$
   (b) **[5 points]** $p \oplus p \oplus p = p$
   (c) **[5 points]** $p \to q \ \equiv\ \neg q \to \neg p$
   (d) **[5 points]** $p \to (q \lor \neg r) \ \equiv\ (p \land r) \to q$

5. Convert the following either to English sentences, if written using propositional logic, or to propositional logic, if written in English. Try to avoid using English phrases, like "if" and "then" in your formal logic, and instead rely on symbols, like $\implies$.
   (a) **[3 points]** If the square of an integer is greater than 9, then that integer is greater than 3 or it is less than -3.
   (b) **[3 points]** For every integer, there exists an integer that is less than that integer.
   (c) **[3 points]** Every real number is a complex number.
   (d) **[3 points]** $(\forall x, y \in \mathbb{Z})(x > 1 \land y > 1) \implies \neg[(\exists z \in \mathbb{Z})(x^3 + y^3 = z^3)]$
   (e) **[4 points]** $(\forall x, y \in \mathbb{R})(x < y) \implies (\exists r \in \mathbb{R})(x < r < y)$
   (f) **[4 points]** $(\forall x \in \mathbb{Z})[x > 1 \implies (\exists p, q \in Z)(p + q = 2x \land \mathsf{Prime}(p) \land \mathsf{Prime}(q))]$

6. Which of the following assertions are true or false for the proposition $P(x, y)$. For any false assertion, provide a counterexample statement for which the assertion is false. For any true example, provide a brief explanation for why it is true.
   (a) **[3 points]** $\exists x \exists y P(x, y) \equiv \exists y \exists x P(x, y)$
   (b) **[3 points]** $\exists x \exists y P(x, y) \implies \exists y \exists x P(x, y)$
   (c) **[3 points]** $\exists x \forall y P(x, y) \equiv \forall y \exists x P(x, y)$
   (d) **[3 points]** $\exists x \forall y P(x, y) \implies \forall y \exists x P(x, y)$
   (e) **[3 points]** $\exists x \forall y P(x, y) \implies \exists y \exists x P(x, y)$