CSCI 1311: Problem Set 2

Due: 10 Feb. 2020

Instructions:

- Your submission must be **typed** and submitted to gradescope as a single pdf.
- You must include a cover page, that contains your name, the assignment information, the date, and your GW email address. No answers to questions should appear on the cover page.
- Try and organize your submission such that answers to questions (or parts of questions) do not span multiple pages. This will make it much easier to grade. **Ideally, each page will start with a new question (or part of question). See the sample for PS0 for a nice easy formatting.**
- On gradescope, be sure to mark which page your answer to each question (or sub question) is located. Doing so inaccurately could lead to issues with grading.

Question Weighting

Question:	1	2	3	4	5	6	7	8	Total
Points:	10	15	15	15	25	20	25	25	150

Note this is problem set is graded out of 100 points, but there are 150 points possible. Do as many problems as you like to get the score you desire. Grades above 100 are allowed and will be considered bonus.

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1. **[10 points]** Prove that for all square integers, n^2,can be written in the form $n^2 = 4q + 1$ or $n^2 = 4q$ for some integer q.
2. **[15 points]** Prove that for all integers $n > 4$, if n is prime than $n = 6q + 1$ or $n = 6q - 1$ where $q \geq 1$.
3. **[15 points]** Prove that the there does not exists two primes greater than 4 have that have a difference of 3, that is $|p - q| \neq 3$ for all primes p and q greater than 4. (*Hint: There are a few ways to do this, but you can also use the result from the prior question to prove this one*)
4. **[15 points]** Prove that, for all integers $n > 1$ and prime p, $p|n$ if, and only if, $p|n^2$.
5. **[25 points]** Prove that $\sqrt{7}$ is irrational.
(*Hint: you can use the result from the previous question to prove this one*)
6. **[20 points]** Using induction, prove that for all integers $n \geq 1$, the inequality $(1 + x)^n \geq 1 + nx$ holds for all integers $x \geq -1$
7. **[25 points]** Using induction, prove that postage of 6-cents or more can be achieved by using only 2-cent and 7-cent stamps.

8. **[25 points]** Using induction, prove that all integers $n \geq 0$, we can write n in base-8. Or put another way, for all integers $n \geq 0$, there exists integers $k \geq 0$ and $d_0 \cdots d_{k-1}$, where each $0 \leq d_i < 8$, such that

$$n = d_{k-1} \cdot 8^{k-1} + d_{k-2} \cdot 8^{k-2} + \ldots + d_1 \cdot 8^1 + d_0 \cdot 8^0.$$

To see why this formulation relates to base-8, let's look at an example. Suppose we wanted to write the base-10 number 245 in base 8. That's the same as

$$245 = \mathbf{3} \cdot 8^2 + \mathbf{6} \cdot 8^1 + \mathbf{5} \cdot 8^0$$

and the base-8 number would be written $365_8 = 245_{10}$, where the subscript indicates the base.