CSCI 1311: Problem Set 4

Due: Mar. 25, 2020

Instructions:

- Your submission must be **typed** and submitted to gradescope as a single pdf.
- You must include a cover page, that contains your name, the assignment information, the date, and your GW email address. No answers to questions should appear on the cover page.
- Try and organize your submission such that answers to questions (or parts of questions) do not span multiple pages. This will make it much easier to grade. **Ideally, each page will start with a new question (or part of question). See the sample for PS0 for a nice easy formatting.**
- On gradescope, be sure to mark which page your answer to each question (or sub question) is located. Doing so inaccurately could lead to issues with grading.

Question Weighting

Question:	1	2	3	4	5	6	7	Total
Points:	20	20	15	10	20	20	10	115

This problem set is graded out of 100 points, scores above 100 points are allowed and considered bonus.

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1. Consider the following sets:

$$A = \{1, 2, 3\}$$
$$B = \{w, x, y, z\}$$

   (a) Let $f : A \to \mathbb{Z}$ be defined as $f(x) = x^2 + 2x + 1$.
      i. **[2 points]** What is the co-domain of f?
      ii. **[2 points]** What is the preimage of 16?
      iii. **[3 points]** Is the function one-to-one? Provide a one sentence explanation.
      iv. **[3 points]** Is the function onto? Provide a one sentence explanation.

   (b) **[10 points]** How many different functions exists of the form $f : A \to B$? Another way to think about this questions, is how many different ways can you map the elements of A to B such that you get a well-defined function? Provide an explanation of how you calculated this result.

2. For the following functions:
   - prove that they are one-to-one, or provide a counter example.

- prove that they are onto, or provide a counter example.

(**place each part on its own page in your submission**)

(a) **[10 points]** $f : \mathbb{Z} \to \mathbb{Z}$, $f(x) = 2x + 1$

(b) **[10 points]** $f : \mathbb{R} \to \mathbb{R}$, $f(x) = (x + 1)^2$

3. **[15 points]** Show that $\mathbb{Z}^+ \times \{0, 1, 2\}$ is countable. You do not need a formal proof, but you should make a compelling argument for why the set is countable, that is, why there exists a one-to-one correspondence function between the set and the positive integers.

   (*Hint: Use the quotient remainder theorem to both define a function and argue that it must be a one-to-one correspondence function.*)

4. **[10 points]** Consider the set of infinite binary strings. This all sequences of 1's and 0's that are infinitely long. For example, you can have the number $1101\ldots$ and the number $0111\ldots$ and so on. Where the $\ldots$ indicates that there are infinitely more 1's and 0's that follow. Prove that the set of all infinite binary strings is uncountable. You should provide a formal proof (via proof by contradiction) based on Cantor's diagonalization technique.

5. Is the following relations an equivalences relation? That is, show that it is reflexive, symmetric, and transitive, or provide a counter example in any category.
   Place answers to each part on its own page in your submission

   (a) **[10 points]** Let $A = \mathbb{Z} \times \mathbb{Z}$, and $(x_1, y_1)\, R\, (x_2, y_2) \iff y_1 = y_2$

   (b) **[10 points]** Let W be all 4 digit numbers, and the relation R is such, for all x and y in W, $x\, R\, y$ if, and only if, the sum of the digits of x is equal to the sum of the digits in y.

6. Are the following partial order relations? That is, show that it is reflexive, antisymmetric, and transitive, or provide a counter example in any category.
   Place answers to each part on its own page in your submission

   (a) **[10 points]** Let R be a relation on the set $\mathbb{Z}$, where $a\, R\, b$ if, and only if, $a \geq b$.

   (b) **[10 points]** Let R be a relation on the set $\mathbb{R}$, where $a\, S\, b$ if, and only if, $x^2 \leq y^2$.

7. **[10 points]** Consider the partial order relation R on $\mathbb{Z}^+ \times \mathbb{Z}^+$ where

   $$(a, b)\, R\, (c, d) \iff a < c \lor (a = c \land b \leq d)$$

   is this a total order relation? Prove your result, or provide a counter example.
   (*Hint: consider using a proof by cases to show your result, or find a counter example*)