CSCI 1311: Problem Set 7

Due: Apr. 29, 2020

Instructions:

Your submission must be **typed** and submitted to gradescope as a single pdf. You **may not** submit handwritten pictures via gradescope.

- You must include a cover page, that contains your name, the assignment information, the date, and your GW email address. No answers to questions should appear on the cover page.
- Try and organize your submission such that answers to questions (or parts of questions) do not span multiple pages. This will make it much easier to grade. **Ideally, each page will start with a new question (or part of question). See the sample for PS0 for a nice easy formatting.**
- On gradescope, be sure to mark which page your answer to each question (or sub question) is located. Doing so inaccurately could lead to issues with grading.

Question Weighting

Question:	1	2	3	4	5	6	Total
Points:	10	30	20	20	15	10	105

This problem set is graded out of 100 points, scores above 100 points are allowed and considered bonus.

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1. **[10 points]** Consider a boolean algebra $(B, +, \cdot, ', 1, 0)$. Define a relation over B as

$$(\forall x, y \in B)(x \, R \, y \iff x \cdot y = x)$$

   Prove that R is a partial order relation and a total order relation.

2. Consider a gumball machine that accepts nickels (5 cents) and dimes (10 cents). A gumball costs 20 cents. Before issuing a gumball, the machine can be in four different states dependent on how much money has been inserted, either 0, 5, 10, or 15 cents. We can encode that state with two Boolean variables, x and y

x	y	state
0	0	0 cents
0	1	5 cents
1	0	10 cents
1	1	15 cents

Let c be boolean variables indicating the next coin input to the gumball machine, c is a 1 if it is a dime, and c is a 0 if it's a nickel.

Provide the boolean formulas for each question part below in DNF. You should also provide a truth table to support your conclusion. It will be useful in later problems.

(a) **[10 points]** What is the boolean formula $g(x, y, c)$ that returns 1 if a gumball should be issued and 0 if not.

(b) **[10 points]** What is the boolean formula $h(x, y, c)$ that returns 1 if a nickel change should be issued or 0 if not.

(c) **[10 points]** Consider **two** boolean functions $f_1(x, y, c = x_{\text{next}}$ and $f_2(x, y, c) = y_{\text{next}}$ where x_{next} and y_{next} describe the *next* x and y variables tracking the state for how much money has been inserted into the machine. ***If the machine issues the gumball (perhaps with change), the state should reset to 0 cents.***

3. Referring back to the gumball machine problem above, using a Karnaugh map to reduce each DNF formula. You must show your work for full credit.

   You may hand draw diagrams, but they need to be inserted into the PDF for submission

   (a) **[5 points]** $g(x, y, c)$

   (b) **[5 points]** $h(x, y, c)$

   (c) **[5 points]** $f_1(x, y, c)$

   (d) **[5 points]** $f_2(x, y, c)$

4. Referring back to the gumball machine example, draw the digital logic circuits for the Boolean formulas below using the reduced expression found using the Karnaugh map

   (a) **[5 points]** $g(x, y, c)$

   (b) **[5 points]** $h(x, y, c)$

   (c) **[5 points]** $f_1(x, y, c)$

   (d) **[5 points]** $f_2(x, y, c)$

5. For each of the following expression, what is the expression in CNF? You should create a truth table to support your answer.

   (a) **[5 points]** $x'yz + x'z' + xyz'$

   (b) **[5 points]** $y'z + x'z + x'z'$

   (c) **[5 points]** $x'yz' + xy'zw + x'y'z'w' + xy'w' + y'zw + xyz$

6. **[10 points]** Use a Karnaugh map to find a reduce form of the expression in 5c (above)